Evoluten und Evolventen

(Anleitung zu einem Kleinprojekt)

Verwende die Ergebnisse des Kleinprojekts "Schläuche":

Rechnung

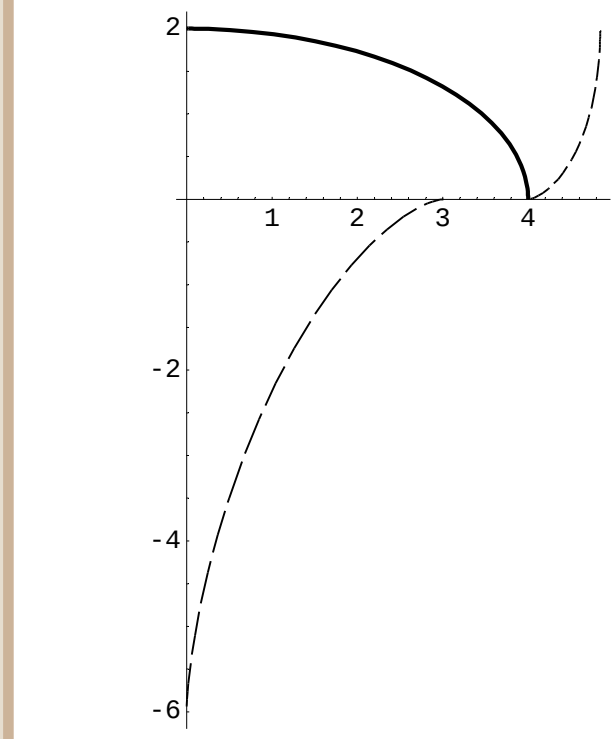
In[1]:=
```
Remove["Global`*"];
```

In[2]:=
```
from[uhu_] := Module[{}, (*Rechnung*)
  v[t_] := {x[t], y[t], 0};
  ϰ[t_] := Sqrt[((v''[t].v''[t]) (v'[t].v'[t]) - (v'[t].v''[t])^2) / ((v'[t
  ρ[t_] := 1 / ϰ[t];
  tT[t_] := v'[t] / Sqrt[v'[t].v'[t]];
  nN[t_] := 1 / ϰ[t] 1 / Sqrt[v'[t].v'[t]] D[v'[t] / Sqrt[v'[t].v'[t]], t];
  evolu[t_] := v[t] + ρ[t] nN[t] / Sqrt[nN[t].nN[t]];
  evolv[t_, anf_] :=
   v[t] + (-1) tT[t] Evaluate[N[Integrate[Sqrt[v'[ti].v'[ti]] , {ti, anf, t}
  v2[t_] := {v[t][[1]], v[t][[2]]};
  evolu2[t_] := {evolu[t][[1]], evolu[t][[2]]};
  evolv2[t_, anf_] := {evolv[t, anf][[1]], evolv[t, anf][[2]]};
  (*Plot 3 D*)
  p1[anf_, end_] := ParametricPlot[Evaluate[v2[t]], {t, anf, end},
    AspectRatio → Automatic, PlotStyle → {Thickness[.01]}, DisplayFunction →
  p2[anf_, end_] := ParametricPlot[Evaluate[evolu2[t]], {t, anf, end}, AspectRatio
    PlotStyle → {Thickness[.005], Dashing[{0.1, 0.02}]}, DisplayFunction → Identit
  p3[anf_, end_] := ParametricPlot[Evaluate[evolv2[t, anf]],
    {t, anf, end}, AspectRatio → Automatic,
    PlotStyle → {Thickness[.005], Dashing[{0.05, 0.015}]}, DisplayFunction →
  Show[
   p1[anf, end],
   p2[anf, end],
   p3[anf, end], DisplayFunction → $DisplayFunction]
 ]
```

Demos Kurvendefinitionen

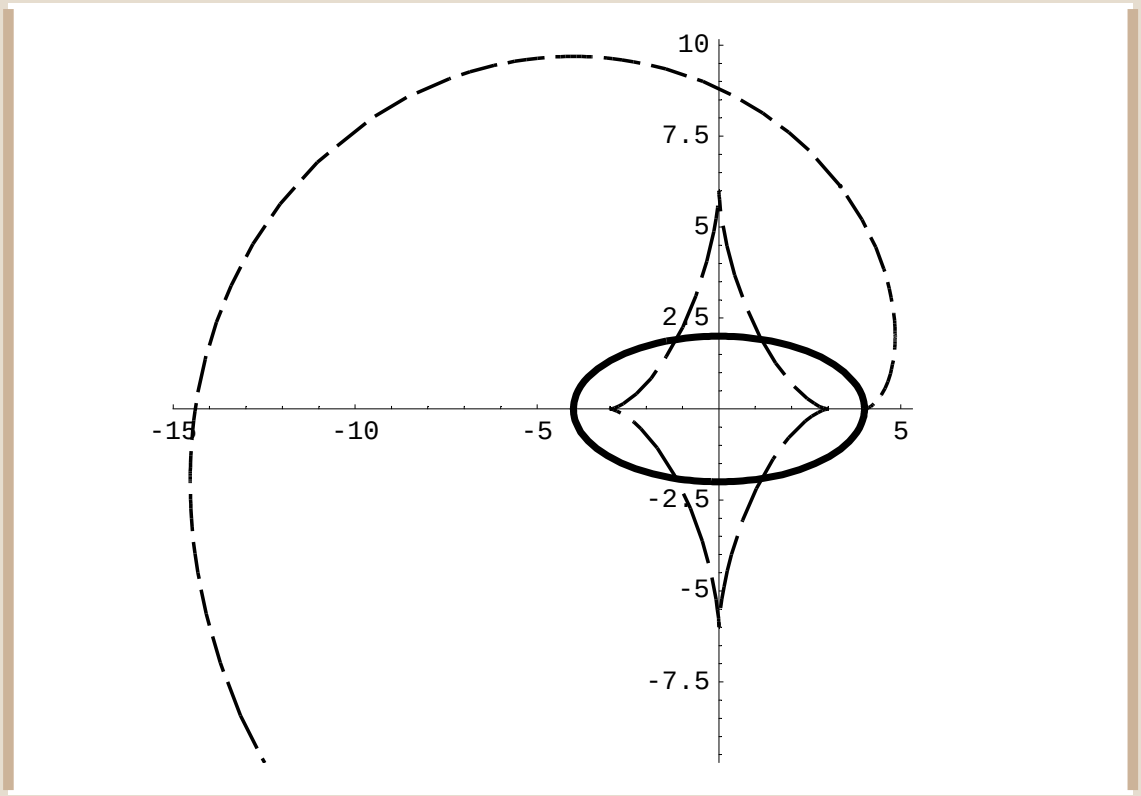
1

In[3]:=
```
(*Kurvendefinitionen*)
x[t_] := 4 Cos[t];
y[t_] := 2 Sin[t];
z[t_] := 0;
anf = 0;
end = Pi / 2;
from[uhu];
```



2

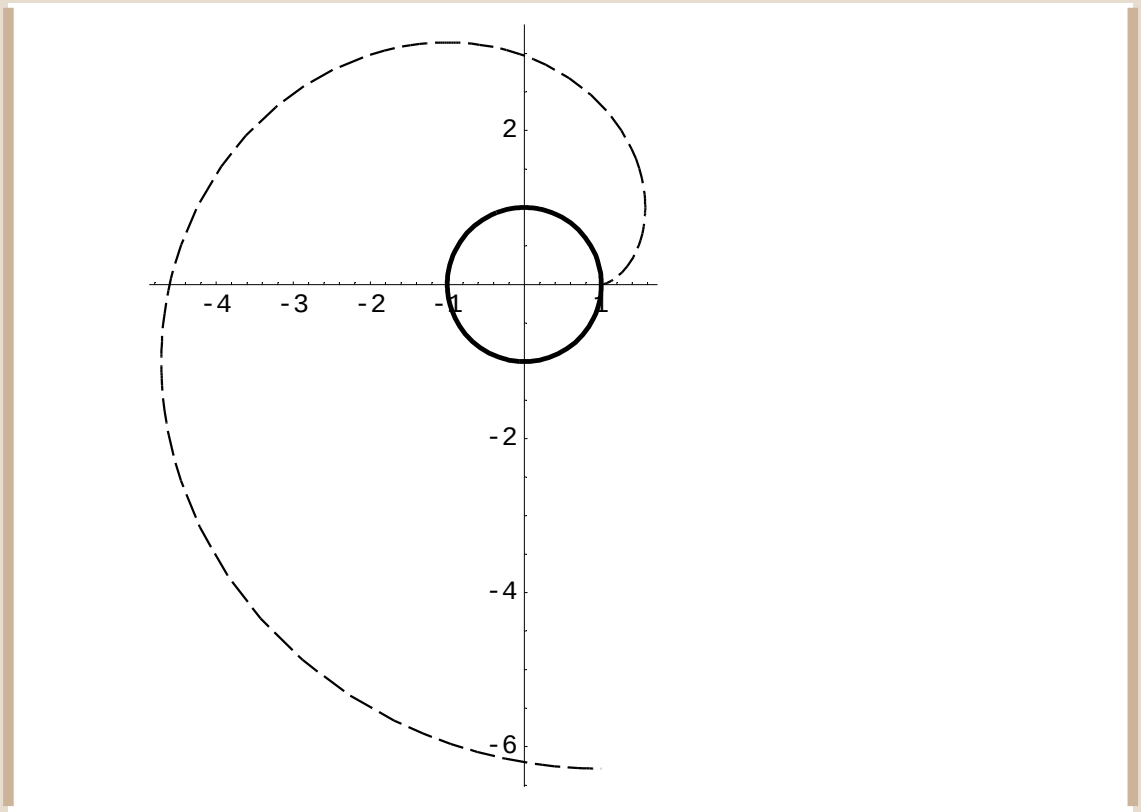
In[9]:=
```
(*Kurvendefinitionen*)
x[t_] := 4 Cos[t];
y[t_] := 2 Sin[t];
z[t_] := 0;
anf = 0;
end = 2 Pi;
from[uhu];
```



3

In[15]:=

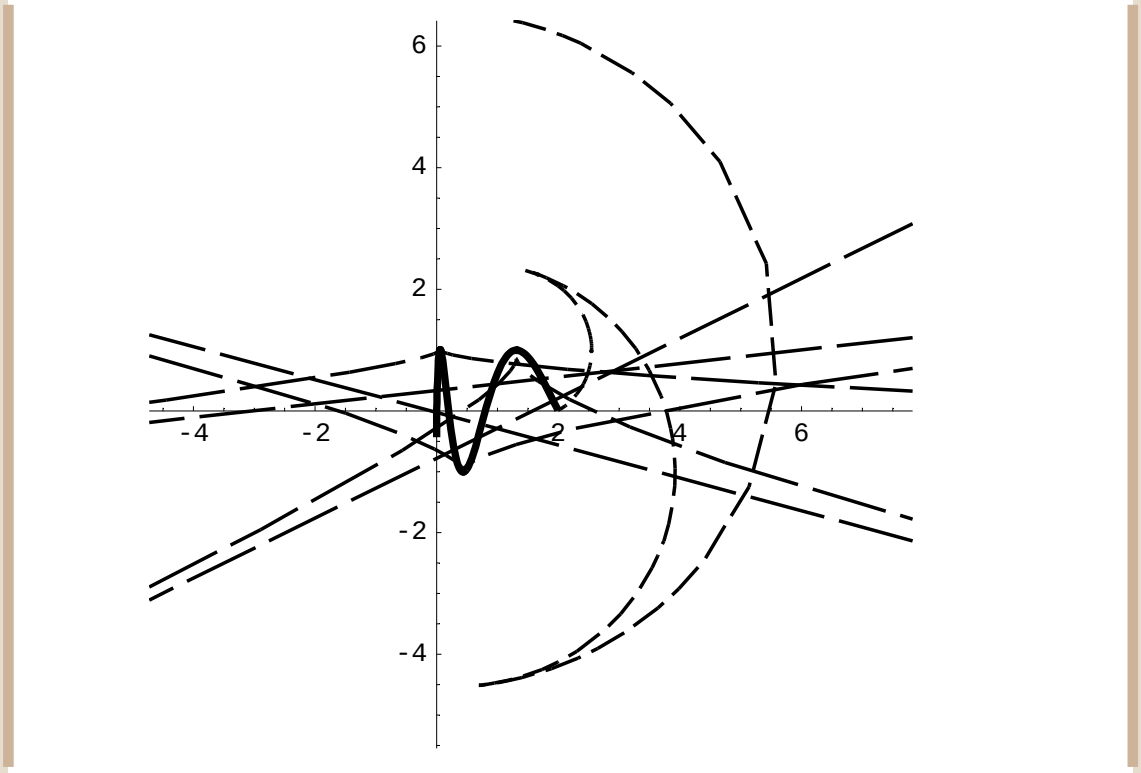
```
(*Kurvendefinitionen*)
x[t_]:= Cos[t];
y[t_]:= Sin[t];
z[t_]:= 0;
anf=0;
end =2Pi;
from[uhu];
```



4

In[22]:=

```
(*Kurvendefinitionen*)
x[t_]:= 1+Cos[t];
y[t_]:= Sin[t^2];
z[t_]:= 0;
anf=0.1;
end =Pi;
from[uhu];
```



Ergebnisse von Untersuchungen der Klasse M2p: Schlauchdefinitionen für interessante Formen

In[29]:=

Rechnung mit Zwischenergebnissen

In[29]:=

```
Remove["Global`*"];
```

In[30]:=

```
from[uhu_] := Module[{}, (*Rechnung*)
   v[t_] := {x[t], y[t], 0};
   Print["v[t]  ", v[t]];
   ϰ[t_] := Sqrt[((v''[t].v''[t]) (v'[t].v'[t]) - (v'[t].v''[t])^2) / ((v'[t
   ρ[t_] := 1 / ϰ[t];
   tT[t_] := v'[t] / Sqrt[v'[t].v'[t]];
   Print["tT[t]  ", tT[t]];
   nN[t_] := 1 / ϰ[t] 1 / Sqrt[v'[t].v'[t]] D[v'[t] / Sqrt[v'[t].v'[t]], t];
   Print["nNT[t]  ", nN[t]];
   evolu[t_] := v[t] + ρ[t] nN[t] / Sqrt[nN[t].nN[t]];
   Print["evolu[t]  ", evolu[t]];
   evolv[t_, anf_] :=
    v[t] + (-1) tT[t] Evaluate[N[Integrate[Sqrt[v'[ti].v'[ti]] , {ti, anf, t}
   Print["evolv[t,anf]  ", evolv[t, anf]];
   v2[t_] := {v[t][[1]], v[t][[2]]};
   Print["v2[t]  ", v2[t]];
   evolu2[t_] := {evolu[t][[1]], evolu[t][[2]]};
   Print["evolu2[t]  ", evolu2[t]];
   evolv2[t_, anf_] := {evolv[t, anf][[1]], evolv[t, anf][[2]]};
   Print["evolv2[t,anf]  ", evolv2[t, anf]];
   (*Plot 3 D*)
   p1[anf_, end_] := ParametricPlot[Evaluate[v2[t]],
       {t, anf, end}, AspectRatio → Automatic, PlotStyle → {Thickness[.02]}];
    p2[anf_, end_] := ParametricPlot[Evaluate[evolu2[t]], {t, anf, end},
       AspectRatio → Automatic, PlotStyle → {Thickness[.01], Dashing[{0.1, 0.02
    p3[anf_, end_] := ParametricPlot[Evaluate[evolv2[t, anf]], {t, anf, end}
       AspectRatio → Automatic, PlotStyle → {Thickness[.01], Dashing[{0.05,
    Show[
     p1[anf, end],
     p2[anf, end],
     p3[anf, end]]
  ];
(*Kurvendefinitionen*)
x[t_] := 4 Cos[t];
y[t_] := 2 Sin[t];
z[t_] := 0;
anf = 0;
end = Pi;
from[uhu];
```

v[t] {4 Cos[t], 2 Sin[t], 0}

tT[t] $\left\{-\frac{4\,\text{Sin}[t]}{\sqrt{4\,\text{Cos}[t]^2+16\,\text{Sin}[t]^2}},\ \frac{2\,\text{Cos}[t]}{\sqrt{4\,\text{Cos}[t]^2+16\,\text{Sin}[t]^2}},\ 0\right\}$

```
nNT[t]
```

$$\left\{\frac{\frac{48\cos[t]\sin[t]^2}{(4\cos[t]^2+16\sin[t]^2)^{3/2}}-\frac{4\cos[t]}{\sqrt{4\cos[t]^2+16\sin[t]^2}}}{\sqrt{4\cos[t]^2+16\sin[t]^2}\sqrt{\frac{-144\cos[t]^2\sin[t]^2+(16\cos[t]^2+4\sin[t]^2)(4\cos[t]^2+16\sin[t]^2)}{(4\cos[t]^2+16\sin[t]^2)^3}}},\right.$$

$$\left.\frac{-\frac{24\cos[t]^2\sin[t]}{(4\cos[t]^2+16\sin[t]^2)^{3/2}}-\frac{2\sin[t]}{\sqrt{4\cos[t]^2+16\sin[t]^2}}}{\sqrt{4\cos[t]^2+16\sin[t]^2}\sqrt{\frac{-144\cos[t]^2\sin[t]^2+(16\cos[t]^2+4\sin[t]^2)(4\cos[t]^2+16\sin[t]^2)}{(4\cos[t]^2+16\sin[t]^2)^3}}},0\right\}$$

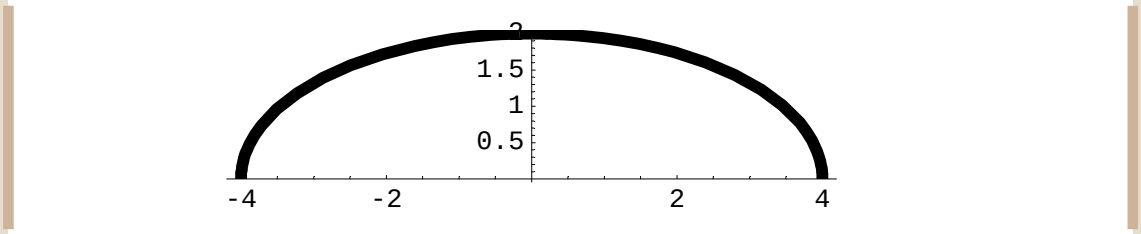
```
evolu[t]  {4 Cos[t] + ((4 Cos[t]^2 + 16 Sin[t]^2)^(5/2)
     (48 Cos[t] Sin[t]^2/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])) /
   ((-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))
    Sqrt[((4 Cos[t]^2 + 16 Sin[t]^2)^2 (48 Cos[t] Sin[t]^2/(4 Cos[t]^2+16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2+16 Sin[t]^2])^2)/
       (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2)) +
      ((4 Cos[t]^2 + 16 Sin[t]^2)^2 (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2+16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2+16 Sin[t]^2])^2)/
       (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))]),
  2 Sin[t] + ((4 Cos[t]^2 + 16 Sin[t]^2)^(5/2)
     (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])) /
   ((-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))
    Sqrt[((4 Cos[t]^2 + 16 Sin[t]^2)^2 (48 Cos[t] Sin[t]^2/(4 Cos[t]^2+16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2+16 Sin[t]^2])^2)/
       (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2)) +
      ((4 Cos[t]^2 + 16 Sin[t]^2)^2 (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2+16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2+16 Sin[t]^2])^2)/
       (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))]),
  0}
```

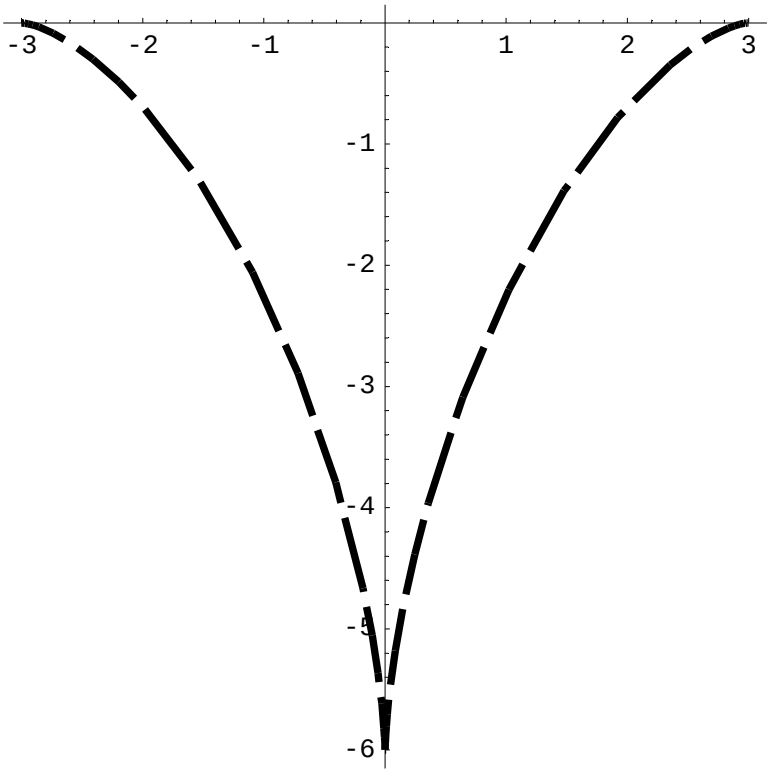
```
evolv[t,anf]  {4 Cos[t] + 8. EllipticE[t, -3.] Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2],
  2 Sin[t] - 4. Cos[t] EllipticE[t, -3.]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2], 0}
```

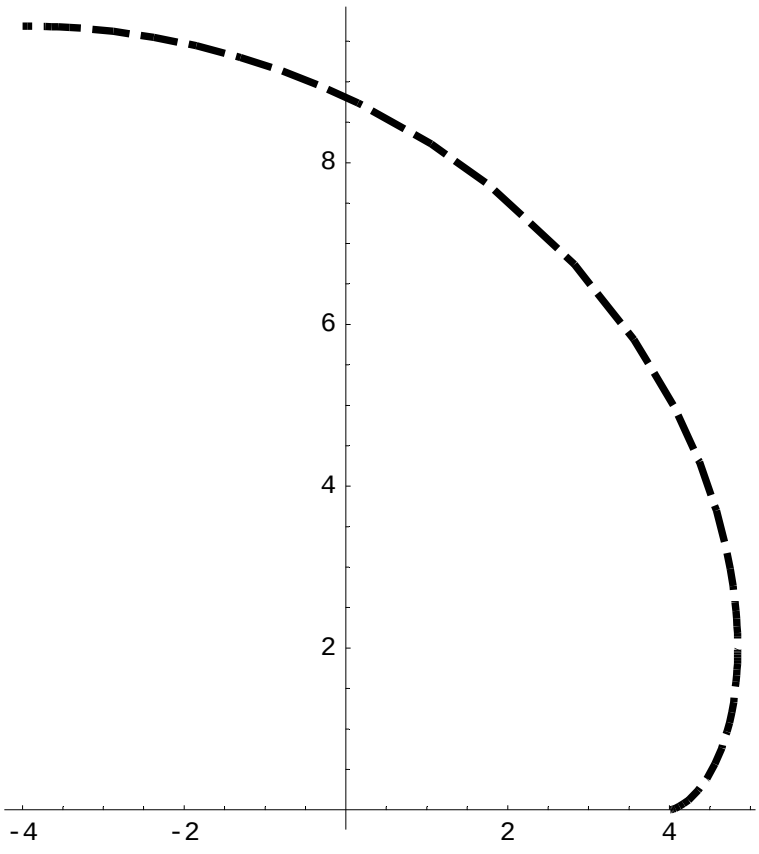
```
v2[t]  {4 Cos[t], 2 Sin[t]}
```

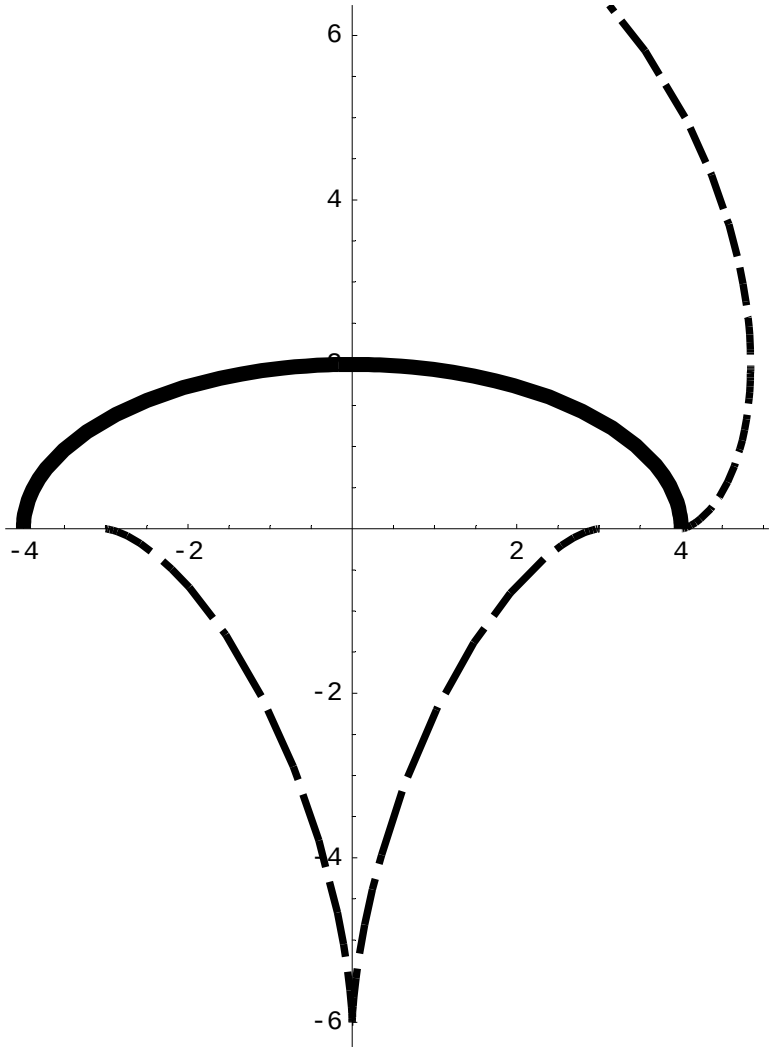
```
evolu2[t_] := {4 Cos[t] + ((4 Cos[t]^2 + 16 Sin[t]^2)^(5/2)
    (48 Cos[t] Sin[t]^2/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])) /
  ((-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))
   Sqrt[(4 Cos[t]^2 + 16 Sin[t]^2)^2 (48 Cos[t] Sin[t]^2/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])^2/
     (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2)) +
    (4 Cos[t]^2 + 16 Sin[t]^2)^2 (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])^2/
     (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))]),
  2 Sin[t] + ((4 Cos[t]^2 + 16 Sin[t]^2)^(5/2)
    (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])) /
  ((-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))
   Sqrt[(4 Cos[t]^2 + 16 Sin[t]^2)^2 (48 Cos[t] Sin[t]^2/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 4 Cos[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])^2/
     (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2)) +
    (4 Cos[t]^2 + 16 Sin[t]^2)^2 (-24 Cos[t]^2 Sin[t]/(4 Cos[t]^2 + 16 Sin[t]^2)^(3/2) - 2 Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2])^2/
     (-144 Cos[t]^2 Sin[t]^2 + (16 Cos[t]^2 + 4 Sin[t]^2) (4 Cos[t]^2 + 16 Sin[t]^2))])}

evolv2[t,anf] := {4 Cos[t] + 8. EllipticE[t, -3.] Sin[t]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2],
  2 Sin[t] - 4. Cos[t] EllipticE[t, -3.]/Sqrt[4 Cos[t]^2 + 16 Sin[t]^2]}
```




-3
-2
-1
1
2
3
-1
-2
-3
-4
-5
-6


8
6
4
2
-4
-2
2
4


6
4
-4
-2
2
4
-2
-4
-6