Umgang mit der Klothoide

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Klothoide

1. Putzmaschine

```
In[1]:= Remove["Global`*"]
```

2. Definitionen

Drehmatrix

```
In[2]:= dreh[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}};
        dreh[φ] // MatrixForm
```

```
Out[3]//MatrixForm=
        ( Cos[φ]   -Sin[φ] )
        ( Sin[φ]    Cos[φ] )
```

Kurven

```
In[4]:= p2[m3_, m4_] = {m3, m4};
```

```
In[5]:= klotVec[len_, a_, c_] :=
          a {Integrate[Cos[c t^2], {t, 0, len}], Integrate[Sin[c t^2], {t, 0, len}]};
```

```
In[6]:= p1[m1_, m2_] = {m1, m2};
```

```
In[7]:= klotVec[len_, a_, c_, p1_] :=
          a {Integrate[Cos[c t^2], {t, 0, len}], Integrate[Sin[c t^2], {t, 0, len}]} + p1;
```

```
In[8]:= klotVec[len_, a_, c_, m1_, m2_] :=
          a {Integrate[Cos[c t^2], {t, 0, len}], Integrate[Sin[c t^2], {t, 0, len}]} + {m1, m2};
```

```
In[9]:= kreis[r_, α_] := r {Cos[α], Sin[α]};
```

```
In[10]:= kreis[r_, α_, m3_, m4_] := r {Cos[α], Sin[α]} + {m3, m4};
```

```
In[11]:= kreis[r_, α_, p2_] := r {Cos[α], Sin[α]} + p2;
```

Plotfunktionen (Beispiele siehe unter Tests und Plots)

```
In[12]:= pKloth[a_, c_, m1_, m2_, len1_, len2_, φ_] :=
          ParametricPlot[Evaluate[dreh[φ].klotVec[len, a, c, m1, m2]],
           {len, len1, len2}, AspectRatio → Automatic]
```

```
In[13]:= pKreis[r_, m3_, m4_, φ1_, φ2_] := ParametricPlot[
           Evaluate[kreis[r, α, m3, m4]], {α, φ1, φ2}, AspectRatio → Automatic];

In[14]:= pKreisVoll[r_, m3_, m4_, φ1_, φ2_] :=
          ParametricPlot[Evaluate[kreis[r, α, m3, m4]], {α, φ1, φ2},
           Epilog → {PointSize[0.03], Hue[1], Point[{m3, m4}],   Line[
              {{m3, m4}, kreis[r, φ1, m3, m4]}],   Line[{{m3, m4}, kreis[r, φ2, m3, m4]}] },
           AspectRatio → Automatic, PlotRange → {m4 - r - 0.1 r, m4 + r + 0.1 r}];

In[15]:= pKreisVoll[r_, p3_, φ1_, φ2_] := pKreisVoll[r, p3[[1]], p3[[2]], φ1, φ2];
```

Tangentenvektoren

1. Ableitung

```
In[16]:= D[Evaluate[klotVec[len, 1, 1 / 2]], len]

Out[16]= {Cos[len^2/2], Sin[len^2/2]}
```

1. Ableitung

```
In[17]:= D[Evaluate[klotVec[len, 1, 1 / 2]], {len, 1}]

Out[17]= {Cos[len^2/2], Sin[len^2/2]}
```

2. Ableitung

```
In[18]:= D[Evaluate[klotVec[len, 1, 1 / 2]], {len, 2}]

Out[18]= {-len Sin[len^2/2], len Cos[len^2/2]}
```

Vektor, n-te Ableitung, normiert

```
In[19]:= dKlotVec[laenge_, a_, c_, m1_, m2_, n_] :=
          Evaluate[ D[Evaluate[klotVec[len, a, c, m1, m2], {len, n}]]] /. {len → laenge}

In[20]:= dKlotVecNorm[laenge_, a_, c_, m1_, m2_, n_] :=
          Evaluate[ D[Evaluate[klotVec[len, a, c, m1, m2], {len, n}]] /
             Norm[D[Evaluate[klotVec[len, a, c, m1, m2], {len, n}]]]] /. {len → laenge}
```

Krümmung der Klothoide

Benütze die Kurve klotVec[len,a,c,m1,m2]

```
In[21]:= κ[laenge_, a_, c_, m1_, m2_] :=
          (Evaluate[(D[(klotVec[len, a, c, m1, m2][[1]]), {len, 1}]
                  D[(klotVec[len, a, c, m1, m2][[2]]), {len, 2}] - D[(klotVec[len, a, c, m1,
                      m2][[1]]), {len, 2}] D[(klotVec[len, a, c, m1, m2][[2]]), {len, 1}]) /
                ((D[(klotVec[len, a, c, m1, m2][[1]]), {len, 1}]^2 +
                    D[(klotVec[len, a, c, m1, m2][[2]]), {len, 1}]^2)^(3 / 2))] //
            Simplify) /. {Cos[c len^2]^2 + Sin[c len^2]^2 → 1, len → laenge}
```

Krümmungsradius

```
In[22]:= κRadius[laenge_, a_, c_, m1_, m2_] := 1 / κ[laenge, a, c, m1, m2];
         κRadius[laenge_, a_, c_, p3_] := 1 / κ[laenge, a, c, p3[[1]], p3[[2]]];
```

---

3. Tests und Plots

Koordinaten und Ableitungen

```
In[24]:= Evaluate[klotVec[len, 1, 1 / 2]]
```

$$\text{Out[24]}= \left\{\sqrt{\pi}\ \text{FresnelC}\left[\frac{\text{len}}{\sqrt{\pi}}\right],\ \sqrt{\pi}\ \text{FresnelS}\left[\frac{\text{len}}{\sqrt{\pi}}\right]\right\}$$

```
In[25]:= Evaluate[klotVec[1, 1, 1 / 2, p1[1, 1]]] // N
Out[25]= {1.97529, 1.16371}

In[26]:= Evaluate[klotVec[1, 1, 1 / 2, 1, 1]] // N
Out[26]= {1.97529, 1.16371}

In[27]:= kreis[1, 2, 5, 8] // N
Out[27]= {4.58385, 8.9093}

In[28]:= kreis[1, 2, p2[5, 8]] // N
Out[28]= {4.58385, 8.9093}
```

Vektor, n-te Ableitung

```
In[29]:= dKlotVec[len, 1, 1 / 2, 10, 20, n]
```

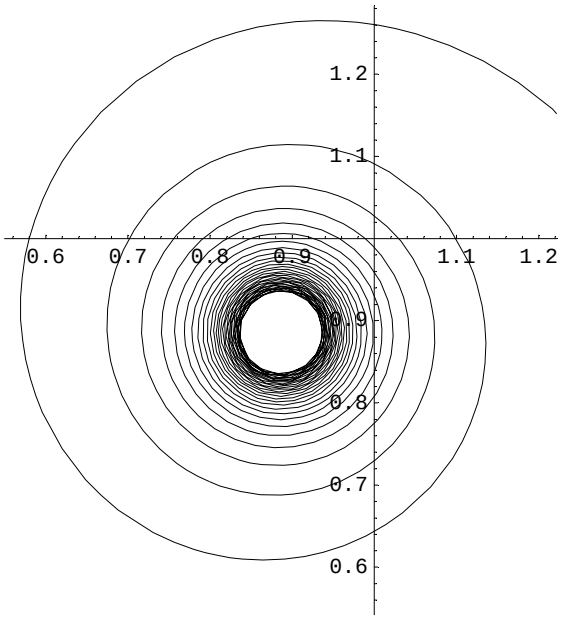
$$\text{Out[29]}= \partial_{\{\text{len},\text{n}\}}\left\{10 + \sqrt{\pi}\ \text{FresnelC}\left[\frac{\text{len}}{\sqrt{\pi}}\right],\ 20 + \sqrt{\pi}\ \text{FresnelS}\left[\frac{\text{len}}{\sqrt{\pi}}\right]\right\}$$

In[30]:= **dKlotVec[laenge, 1, 1 / 2, 10, 20, 3]**

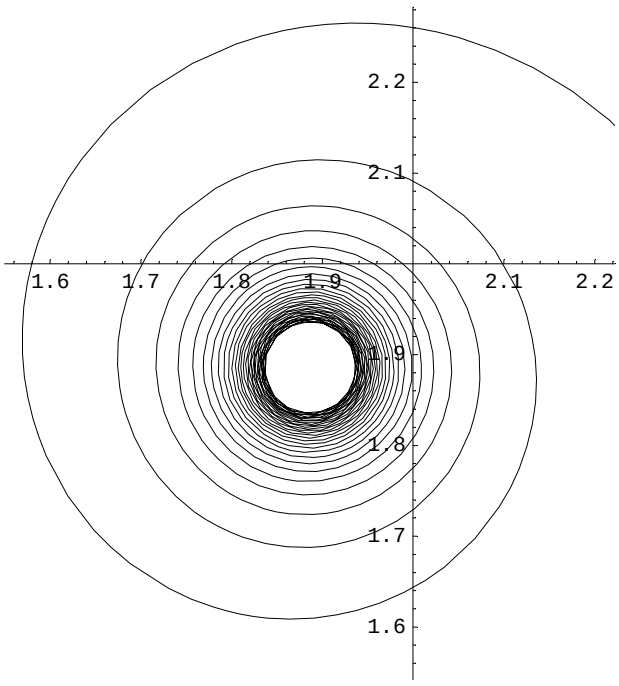
Out[30]= $\left\{\sqrt{\pi}\left(-\frac{\text{laenge}^2\,\text{Cos}\left[\frac{\text{laenge}^2}{2}\right]}{\sqrt{\pi}}-\frac{\text{Sin}\left[\frac{\text{laenge}^2}{2}\right]}{\sqrt{\pi}}\right),\sqrt{\pi}\left(\frac{\text{Cos}\left[\frac{\text{laenge}^2}{2}\right]}{\sqrt{\pi}}-\frac{\text{laenge}^2\,\text{Sin}\left[\frac{\text{laenge}^2}{2}\right]}{\sqrt{\pi}}\right)\right\}$

Klothoidenstücke

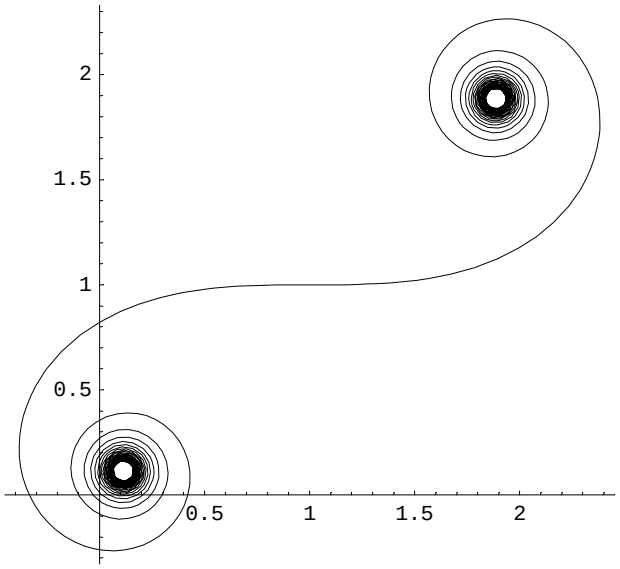
In[31]:=
```
plo1 = ParametricPlot[
    Evaluate[klotVec[len, 1, 1 / 2]], {len, 0, 20}, AspectRatio → Automatic];
```



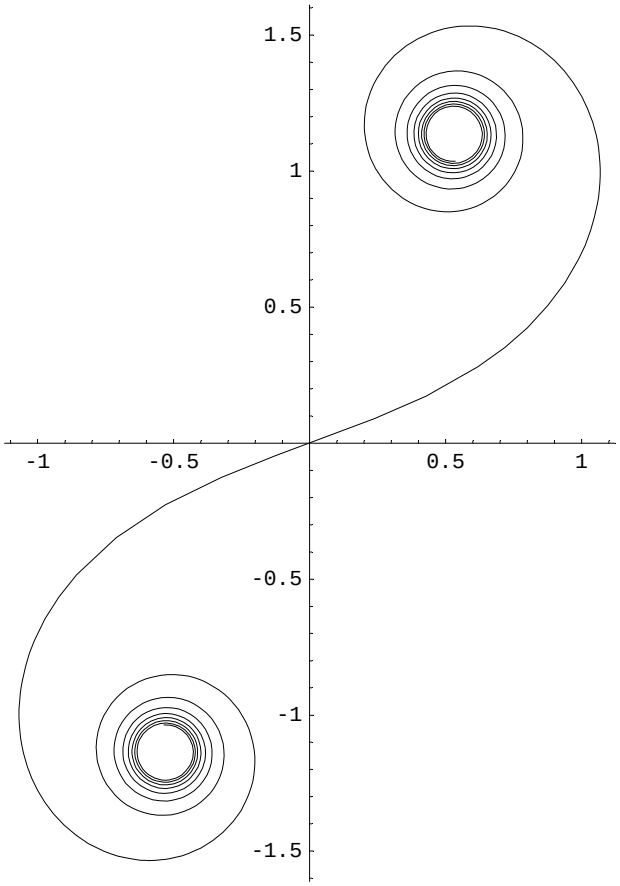
In[32]:=
```
plo2 = ParametricPlot[Evaluate[klotVec[len, 1, 1 / 2, p1[1, 1]]],
    {len, 0, 20}, AspectRatio → Automatic];
```



In[33]:=
```
plo3 = ParametricPlot[Evaluate[klotVec[len, 1, 1 / 2, 1, 1]],
   {len, -20, 20}, AspectRatio → Automatic];
```



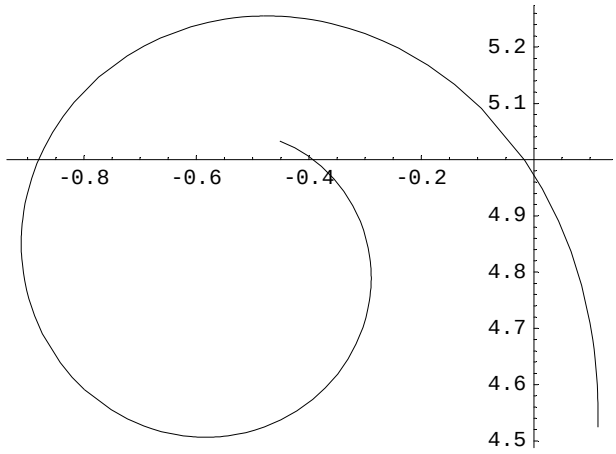
In[34]:=
```
plo4 = ParametricPlot[Evaluate[dreh[Pi / 9].klotVec[len, 1, 1 / 2]],
   {len, -10, 10}, AspectRatio → Automatic]
```



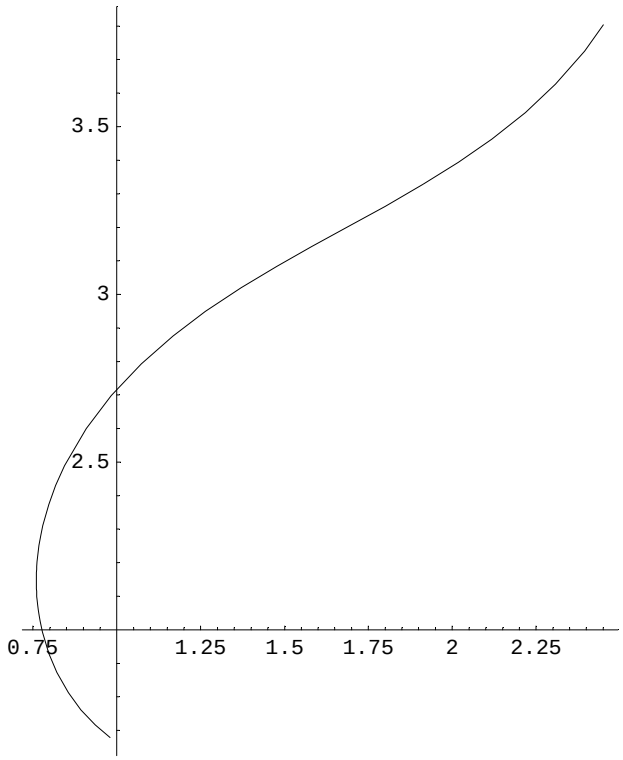
Out[34]= - Graphics -

Klothoidenstücke mit Plot als Funktion

In[35]:= **plo5 = pKloth[1, 1 / 2, 3, 2, 1, 4, Pi / 3];**

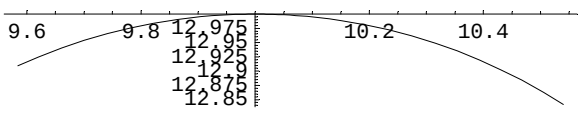


In[36]:= **plo6 = pKloth[1, 1 / 2, 3, 2, 1, -2, 0.5];**

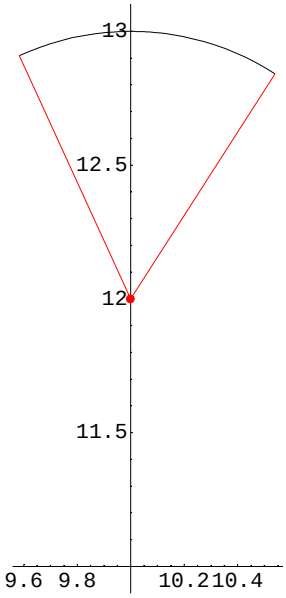


Kreisstücke mit Plot als Funktion

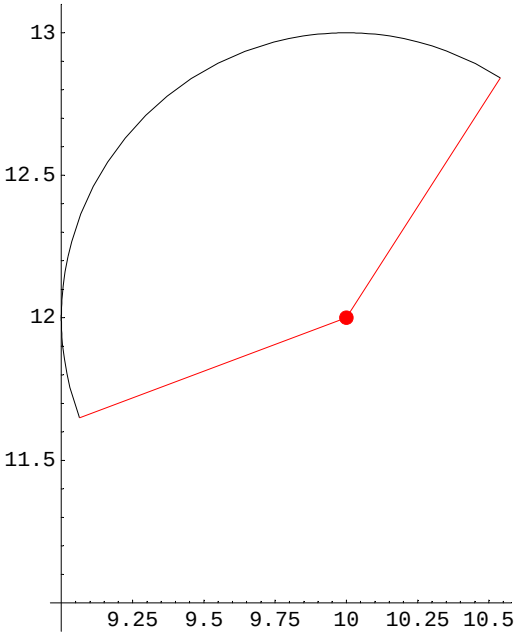
In[37]:= **plo7 = pKreis[1, 10, 12, 1, 2];**



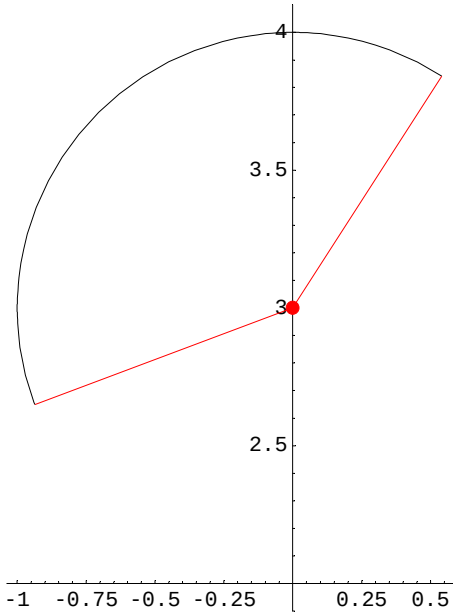
In[38]:= **plo8 = pKreisVoll[1, 10, 12, 1, 2];**



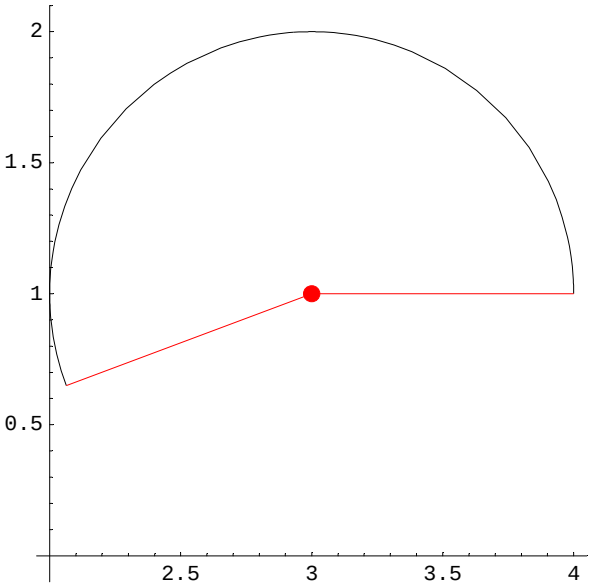
In[39]:= **plo9 = pKreisVoll[1, 10, 12, 1, 3.5];**



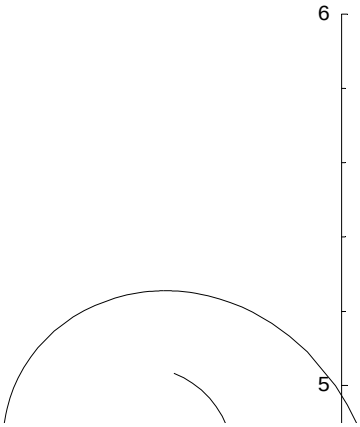
In[40]:= **plo10 = pKreisVoll[1, 0, 3, 1, 3.5];**

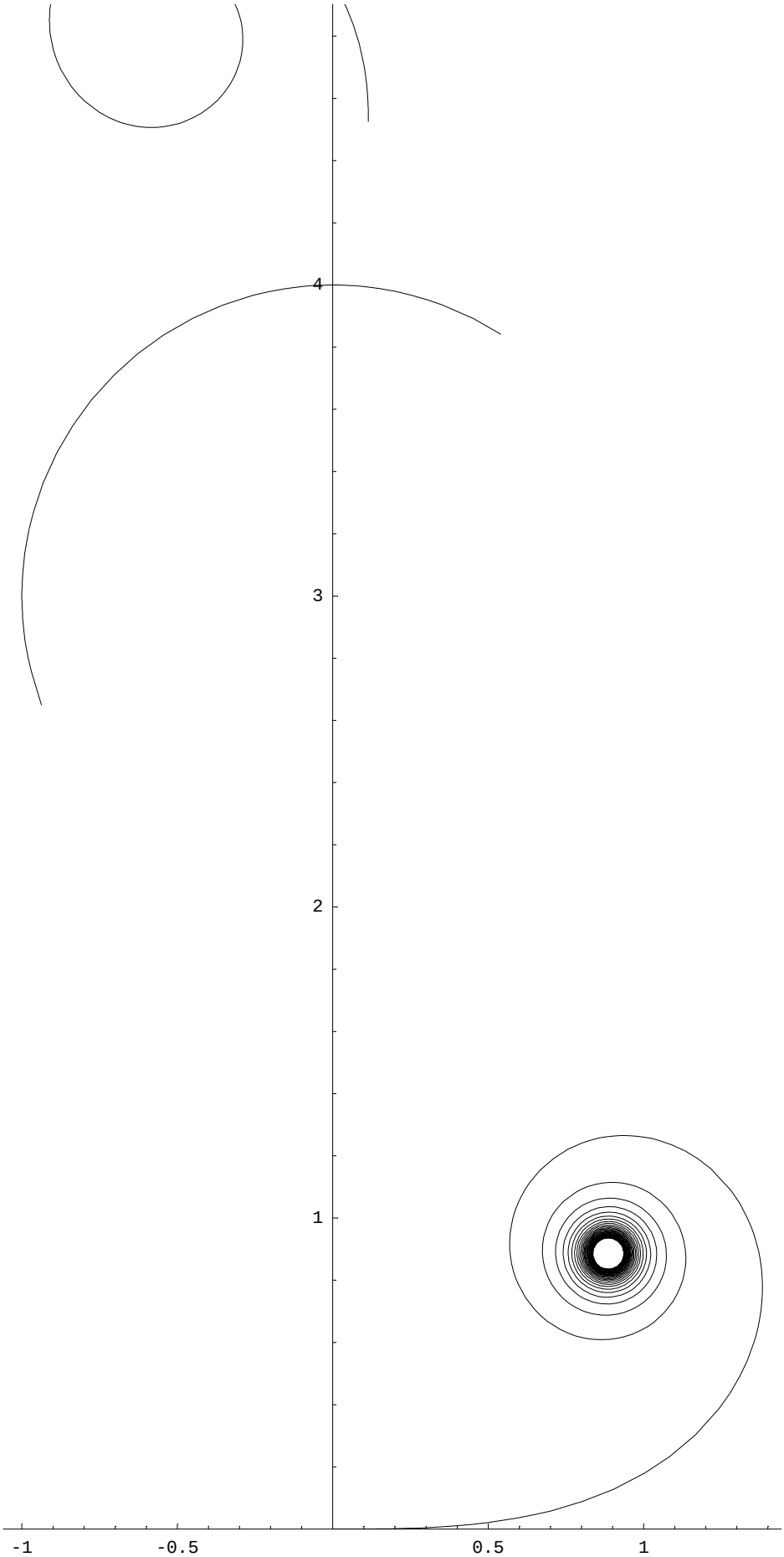


In[41]:= **pKreisVoll[1, {3, 1}, 0, 3.5];**



In[42]:= **Show[plo5, plo1, plo10, PlotRange → {0, 6}];**




4
3
2
1
-1
-0.5
0.5
1

Krümmung und Krümmungsradius der Klothoide

In[43]:= **κ[laenge, a, c, m1, m2]**

Out[43]= $\frac{2\,c\,\text{laenge}}{\sqrt{a^2}}$

In[44]:= **κ[len, 1, 1 / 2, 3, 4]**

Out[44]= len

In[45]:= **κ[20, 1, 1 / 2, 3, 4]**

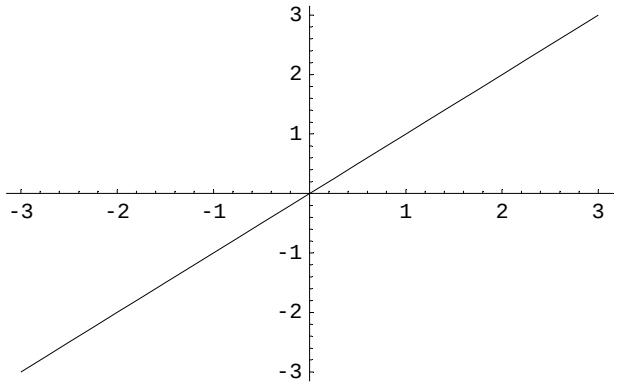
Out[45]= 20

In[46]:= **κRadius[20, 1, 1 / 2, 3, 4]**

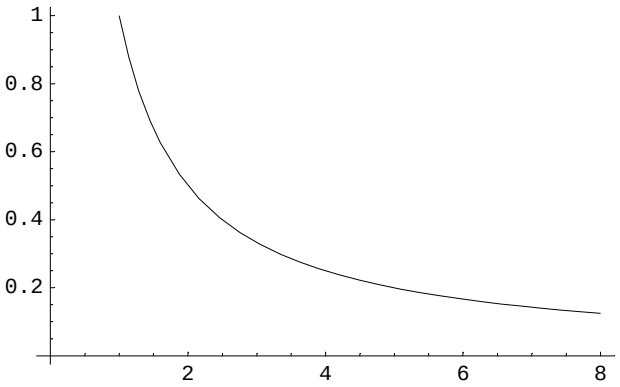
Out[46]= $\frac{1}{20}$

Grösse der Krümmung geplotet

In[47]:= **Plot[Evaluate[κ[len, 1, 1 / 2, 3, 4]], {len, -3, 3}];**

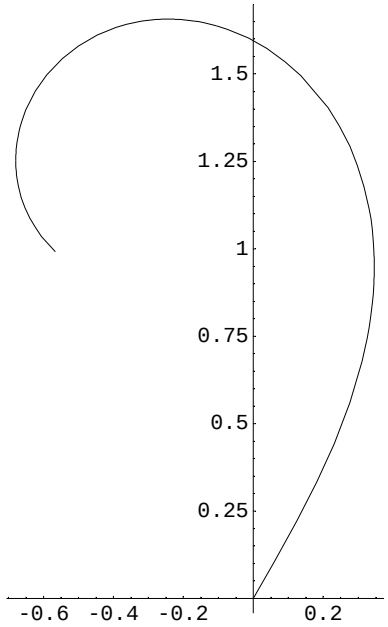


In[48]:= **Plot[Evaluate[κRadius[u, 1, 1 / 2, 3, 4]], {u, 1, 8}];**



4. Eine einfache Zusammensetzung einer Klothoide mit einem Kreis: Der Krümmungsradius des Kreises soll der gleiche wie der der Klothoide sein!

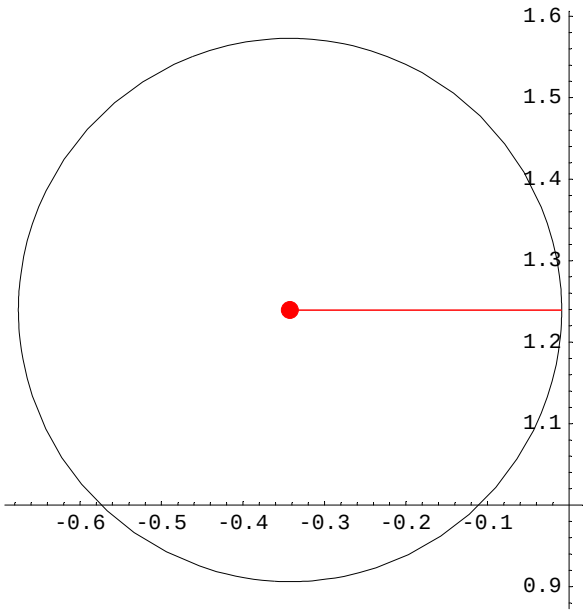
```
In[49]:= (*  pKloth[a_,c_,m1_,m2_,len1_,len2_,φ_]  *)

In[50]:= pl1 = pKloth[1, 1 / 2, 0, 0, 0, 3, Pi / 3];
```




```
In[51]:= (*  dKlotVec[laenge_,a_,c_,m1_,m2_,n_]  *)

In[52]:= tang[0] = Evaluate[dreh[Pi / 3].dKlotVecNorm[0, 1, 1 / 2, 0, 0, 1] // N]

Out[52]= {0.5, 0.866025}

In[53]:= Norm[tang[0]]

Out[53]= 1.

In[54]:= tang[3] = Evaluate[dreh[Pi / 3].dKlotVecNorm[3, 1, 1 / 2, 0, 0, 1] // N]

Out[54]= {0.741168, -0.67132}

In[55]:= Norm[tang[3]]

Out[55]= 1.

In[56]:= (*  klotVec[len_,a_,c_,m1_,m2_]  *)

In[57]:= pun[0] = Evaluate[dreh[Pi / 3].klotVec[0, 1, 1 / 2, 0, 0]]

Out[57]= {0, 0}

In[58]:= pun[3] = Evaluate[dreh[Pi / 3].klotVec[3, 1, 1 / 2, 0, 0] // N]

Out[58]= {-0.565961, 0.99243}
```

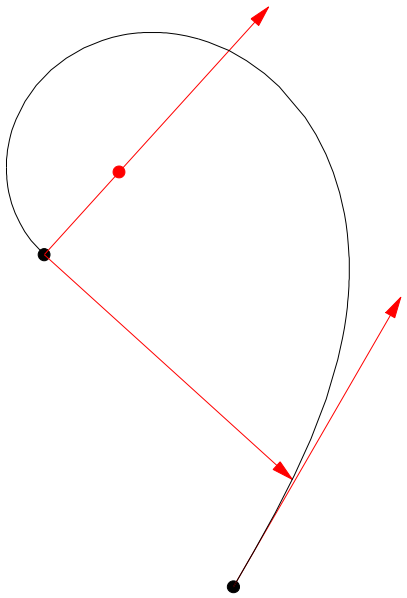
In[59]:=
```
(*   pKreisVoll[r_,p3_,φ1_,φ2_], ϰRadius[laenge_,a_,c_,p3_]*)
```

In[60]:=
```
kr[3] = pKreisVoll[ Evaluate[ϰRadius[3, 1, 1 / 2, pun[3]]],
   pun[3] + dreh[Pi / 2].tang[3]  ϰRadius[3, 1, 1 / 2, pun[3]], 0, 2 Pi];
```

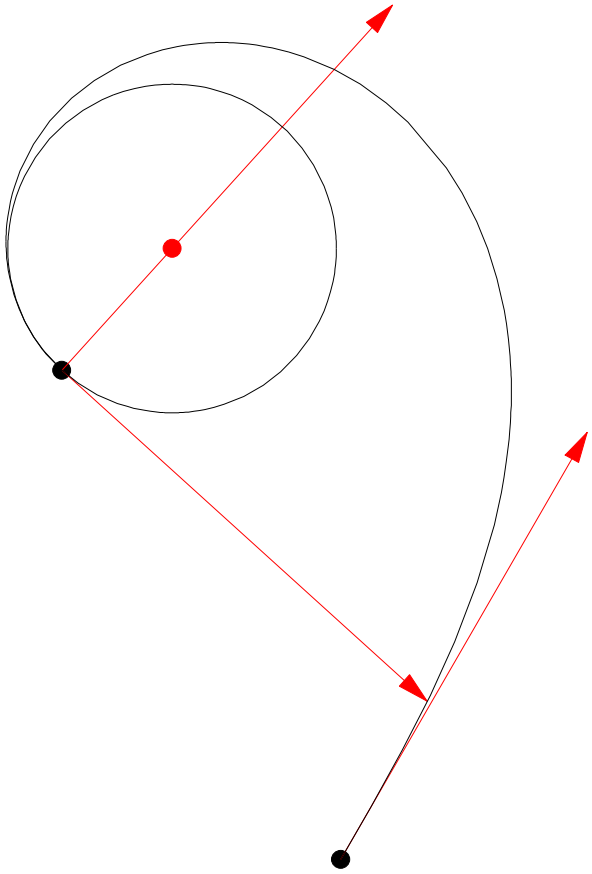


In[61]:=
```
<< Graphics`Arrow`
```

In[62]:=
```
Show[Graphics[{PointSize[0.03], Point[pun[0]], Point[pun[3]],
    Hue[0], Point[pun[3] + dreh[Pi / 2].tang[3]  ϰRadius[3, 1, 1 / 2, pun[3]] ],
    Arrow[pun[0], pun[0] + tang[0]], Arrow[pun[3], pun[3] + tang[3]],
    Arrow[pun[3], pun[3] + dreh[Pi / 2].tang[3]]}], pl1, AspectRatio → Automatic];
```



```
In[63]:= Show[Graphics[{PointSize[0.03], Point[pun[0]], Point[pun[3]],
           Hue[0], Point[pun[3] + dreh[Pi / 2].tang[3]  ×Radius[3, 1, 1 / 2, pun[3]] ],
           Arrow[pun[0], pun[0] + tang[0]],
           Arrow[pun[3], pun[3] + tang[3]], Arrow[pun[3], pun[3] + dreh[Pi / 2].tang[3] ]}],
         pl1, kr[3], AspectRatio → Automatic];
```



4. Weitere Zusammensetzungen von Klothoiden und Kreisen (Strassen- und Schienenbau): Das ist jetzt die praktische Arbeit des Beauftragten...

Weiter mit eigener Arbeit! .....