Lösungen

1

```
Remove["Global´*"]
```

```
EVNorm[Matrix_] :=
 Transpose[Table[N[Transpose[Matrix][[k]] / Norm[Transpose[Matrix][[k]]]],
   {k, 1, Length[Transpose[Matrix]]}]]
```

```
X = {{1, 2, 1}, {1, 2, 2}, {1, 1, 3}};
Dl = {{1, 0, 0}, {0, -1, 0}, {0, 0, -1}};
M = X.Dl.Inverse[X];
MatrixForm[M]
```

$$\begin{pmatrix} 7 & -10 & 4 \\ 8 & -11 & 4 \\ 8 & -10 & 3 \end{pmatrix}$$

```
v1 = {4, -3, 5}; v2 = {-3, 4, 5};
```

- **a**

```
Eigenvalues[M]
```

```
{-1, -1, 1}
```

- **b Eigenvektoren zu doppelten Eigenwerten nicht eindeutig**

```
Eigenvectors[M] // Transpose // MatrixForm
```

$$\begin{pmatrix} -1 & 5 & 1 \\ 0 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

```
EVNorm[Eigenvectors[M] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.447214 & 0.780869 & 0.57735 \\ 0. & 0.624695 & 0.57735 \\ 0.894427 & 0. & 0.57735 \end{pmatrix}$$

- **c**

```
Inverse[M] // MatrixForm
```

$$\begin{pmatrix} 7 & -10 & 4 \\ 8 & -11 & 4 \\ 8 & -10 & 3 \end{pmatrix}$$

```
M == Inverse[M]
```

```
True
```

```
Eigenvalues[Inverse[M]]
```

```
{-1, -1, 1}
```

- **d Eigenvektoren zu doppelten Eigenwerten nicht eindeutig**

```
Eigenvectors[Inverse[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} -1 & 5 & 1 \\ 0 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Inverse[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.447214 & 0.780869 & 0.57735 \\ 0. & 0.624695 & 0.57735 \\ 0.894427 & 0. & 0.57735 \end{pmatrix}$$

e

```
Eigenvalues[Transpose[M]]
```

```
{-1, -1, 1}
```

f

```
Eigenvectors[Transpose[M]] // Transpose // MatrixForm
```

$$\begin{pmatrix} -1 & -1 & 4 \\ 0 & 1 & -5 \\ 1 & 0 & 2 \end{pmatrix}$$

```
EVNorm[Eigenvectors[Transpose[M]] // Transpose] // MatrixForm
```

$$\begin{pmatrix} -0.707107 & -0.707107 & 0.596285 \\ 0. & 0.707107 & -0.745356 \\ 0.707107 & 0. & 0.298142 \end{pmatrix}$$

g

```
w = Cross[v1, v2]
```

```
{-35, -35, 7}
```

```
InhPar = Norm[Cross[v1, v2]]
```

$$7\sqrt{51}$$

```
N[%]
```

```
49.99
```

h

```
InhMPar = Norm[Cross[M.v1, M.v2]]
```

$$77\sqrt{123}$$

```
N[%]
```

```
InhMPar / InhPar
```

$$11\sqrt{\frac{41}{17}}$$

```
N[%]
```

```
17.0828
```

```
Det[M]
```

```
1
```

Kein direkter Einfluss sichtbar.

2

a

```
X = {v1, v2, w} // Transpose; X // MatrixForm
```

$$\begin{pmatrix} 4 & -3 & -35 \\ -3 & 4 & -35 \\ 5 & 5 & 7 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, -2, 0}, {0, 0, 3}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} \frac{509}{357} & \frac{611}{357} & \frac{35}{51} \\ \frac{614}{357} & \frac{359}{357} & -\frac{70}{51} \\ \frac{260}{357} & -\frac{505}{357} & -\frac{22}{51} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 1.42577 & 1.71148 & 0.686275 \\ 1.71989 & 1.0056 & -1.37255 \\ 0.728291 & -1.41457 & -0.431373 \end{pmatrix}$$

```
357 A // MatrixForm
```

$$\begin{pmatrix} 509 & 611 & 245 \\ 614 & 359 & -490 \\ 260 & -505 & -154 \end{pmatrix}$$

b

```
OQStrich = A.(2 v1 - 3 v2)
```

```
{-10, 18, 40}
```

c

```
OQStrichStrich = A.OQStrich
```

```
{44, -54, -50}
```

3

```
Remove["Global`*"]
```

a

```
a = {1, -1, 2}; b = {-2, 1, 4}; w = Cross[a, b]
```

```
{-6, -8, -1}
```

```
X = Transpose[{a, b, w}]; X // MatrixForm
```

$$\begin{pmatrix} 1 & -2 & -6 \\ -1 & 1 & -8 \\ 2 & 4 & -1 \end{pmatrix}$$

```
Dλ = {{1, 0, 0}, {0, 1, 0}, {0, 0, -1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
B = X.Dλ.Inverse[X];  B // MatrixForm
```

$$\begin{pmatrix} \frac{29}{101} & -\frac{96}{101} & -\frac{12}{101} \\ -\frac{96}{101} & -\frac{27}{101} & -\frac{16}{101} \\ -\frac{12}{101} & -\frac{16}{101} & \frac{99}{101} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 0.287129 & -0.950495 & -0.118812 \\ -0.950495 & -0.267327 & -0.158416 \\ -0.118812 & -0.158416 & 0.980198 \end{pmatrix}$$

```
101 B // MatrixForm
```

$$\begin{pmatrix} 29 & -96 & -12 \\ -96 & -27 & -16 \\ -12 & -16 & 99 \end{pmatrix}$$

- **b**

```
B.{5, 4, 2}
```

$$\left\{-\frac{263}{101}, -\frac{620}{101}, \frac{74}{101}\right\}$$

```
N[%]
```

```
{-2.60396, -6.13861, 0.732673}
```

- **c**

```
Dλ^100
```

```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

```
Bhoch100 = X.(Dλ^100).Inverse[X];  Bhoch100 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Bhoch100 == IdentityMatrix[3]
```

```
True
```

4

```
Remove["Global`*"]
```

$$\left(U^{-1}\,W\right)^{-1}\,\left(U^{-1}\,W\right)\,X\,W^{T}-E=\left(\left(\left(U^{-1}\right)^{T}\,W^{T}\right)^{-1}\right)^{T}\;==>\;\left(U^{-1}\,W\right)^{-1}\,\left(U^{-1}\,W\right)=E\ \text{oder}$$

$$==>\quad W^{-1}\,U\;U^{-1}\,W\,X\,W^{T}-E=\left(\left(\left(W\,U^{-1}\right)^{T}\right)^{-1}\right)^{T}=\left(\left(\left(W\,U^{-1}\right)^{T}\right)^{T}\right)^{-1}=\left(W\,U^{-1}\right)^{-1}=U\,W^{-1}$$

$$==>\qquad X\,W^{T}=U\,W^{-1}+E$$

$$==>\qquad X=U\,W^{-1}\left(W^{T}\right)^{-1}+\left(W^{T}\right)^{-1}=U\,\left(W^{T}\,W\right)^{-1}+\left(W^{T}\right)^{-1}$$

5

```
Remove["Global`*"]
```

a

```
OP1 = {5, 0, 1};  OP2 = {4, 1, -1};  OP3 = {7 / 2, 2, 10};
OP4 = {2, 6, 1};  OP5 = {-1, 5, 8};  OP6 = {-2, 12, 0};

G1 = {OP1, OP2, OP3} // Transpose;
G2 = {OP4, OP5, OP6} // Transpose;
Det[G1]
```

$$\frac{129}{2}$$

```
Det[G2]
```

$$-290$$

```
G.G1 == G2
```

$$G.\left\{\left\{5, 4, \frac{7}{2}\right\}, \{0, 1, 2\}, \{1, -1, 10\}\right\} == \{\{2, -1, -2\}, \{6, 5, 12\}, \{1, 8, 0\}\}$$

```
G = G2.Inverse[G1];  G // MatrixForm
```

$$\begin{pmatrix} \frac{16}{43} & -\frac{101}{43} & \frac{6}{43} \\ \frac{140}{129} & \frac{53}{43} & \frac{74}{129} \\ \frac{56}{129} & \frac{219}{43} & -\frac{151}{129} \end{pmatrix}$$

```
129 G // MatrixForm
```

$$\begin{pmatrix} 48 & -303 & 18 \\ 140 & 159 & 74 \\ 56 & 657 & -151 \end{pmatrix}$$

```
G // N // MatrixForm
```

$$\begin{pmatrix} 0.372093 & -2.34884 & 0.139535 \\ 1.08527 & 1.23256 & 0.573643 \\ 0.434109 & 5.09302 & -1.17054 \end{pmatrix}$$

b

```
Dreh[ϕ_] := {{1, 0, 0}, {0, Cos[ϕ], -Sin[ϕ]}, {0, Sin[ϕ], Cos[ϕ]}};  Dreh[ϕ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\phi] & -\text{Sin}[\phi] \\ 0 & \text{Sin}[\phi] & \text{Cos}[\phi] \end{pmatrix}$$

```
Dreh[12 Degree] // N // MatrixForm
```

$$\begin{pmatrix} 1. & 0. & 0. \\ 0. & 0.978148 & -0.207912 \\ 0. & 0.207912 & 0.978148 \end{pmatrix}$$

c

```
OP7 = Dreh[12 Degree].OP1
```

$$\{5, -\text{Sin}[12\,°], \text{Cos}[12\,°]\}$$

```
Dreh[12 Degree].OP1 // N
```

$$\{5., -0.207912, 0.978148\}$$

■ d

G.OP7 // MatrixForm

$$\begin{pmatrix} \frac{80}{43} + \frac{6}{43}\,\text{Cos}\left[12\,^\circ\right] + \frac{101}{43}\,\text{Sin}\left[12\,^\circ\right] \\ \frac{700}{129} + \frac{74}{129}\,\text{Cos}\left[12\,^\circ\right] - \frac{53}{43}\,\text{Sin}\left[12\,^\circ\right] \\ \frac{280}{129} - \frac{151}{129}\,\text{Cos}\left[12\,^\circ\right] - \frac{219}{43}\,\text{Sin}\left[12\,^\circ\right] \end{pmatrix}$$

G.OP7 // N // MatrixForm

$$\begin{pmatrix} 2.4853 \\ 5.7312 \\ -0.0333199 \end{pmatrix}$$