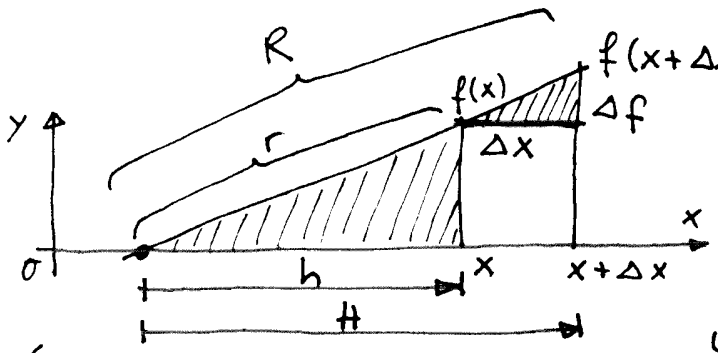
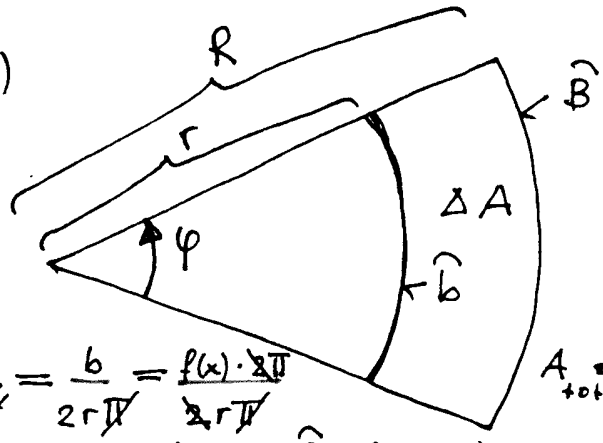
Oberfläche eines Rotationskörpers





$A_{tot} = ?$

(1) $\frac{\varphi}{2\pi} = \frac{b}{2r\pi} = \frac{f(x)\cdot 2\pi}{2r\pi}$

$\hat{B} = f(x+\Delta x)\cdot 2\pi$

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$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{f(x)}{h} = \frac{f(x+\Delta x)}{H}$ (2)

$R^2 = H^2 + f(x+\Delta x)^2$ (3)

$r^2 = h^2 + f(x)^2$ (4)

$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$ $\qquad g(x) \xrightarrow[\Delta x \to 0]{} 0$

$\Rightarrow \Delta f = f'(x)\cdot\Delta x + \underbrace{g(x)\Delta x}_{\hookrightarrow 0\cdot 0} \approx f'(x)\cdot\Delta x$ (5)

$\Delta A = (R^2\cdot\pi - r^2\cdot\pi)\cdot\frac{\varphi}{2\pi} = (R^2-r^2)\cdot\frac{\varphi}{2}$

$h \overset{(2)}{=} \frac{f(x)}{\Delta f}\cdot\Delta x \overset{(5)}{\approx} \frac{f(x)\cdot\cancel{\Delta x}}{f'(x)\cdot\cancel{\Delta x}} = \frac{f(x)}{f'(x)}$, $\quad H \overset{(2)}{=} \frac{f(x+\Delta x)}{\Delta f}\cdot\Delta x \overset{(5)}{\approx} \frac{f(x+\Delta x)\cdot\cancel{\Delta x}}{f'(x)\cdot\cancel{\Delta x}} = \dots$ (6)

$\Rightarrow \Delta A = (R^2-r^2)\frac{\varphi}{2} \overset{(1,3,4)}{=} \left(H^2 + f(x+\Delta x)^2 - h^2 - f(x)^2\right)\cdot\frac{f(x)\cdot\cancel{2}}{\cancel{2}\,r}\cdot\pi$

$\overset{(4),(6)}{=} \left(\frac{f(x+\Delta x)^2}{f'(x)^2} + f(x+\Delta x)^2 - \frac{f(x)^2}{f'(x)^2} - f(x)^2\right)\cdot\frac{f(x)\cdot\pi}{\sqrt{h(x)^2+f(x)^2}}$

$\overset{(6)}{=} \left(f(x+\Delta x)^2 - f(x)^2\right)\cdot\left(\frac{1}{f'(x)^2}+1\right)\cdot\frac{f(x)\cdot\pi}{\sqrt{\frac{f(x)^2}{f'(x)^2}+f(x)^2}}$

$\overset{(5)}{=} \underbrace{\left(\left(f(x)+f'(x)\cdot\Delta x\right)^2 - f(x)^2\right)}\left(\frac{1}{f'(x)^2}+1\right)\cdot\frac{\cancel{f(x)}\cdot\pi}{\cancel{f(x)}\cdot\sqrt{\frac{1}{f'(x)^2}+1}}$

$= \left(\cancel{f(x)^2} + 2f(x)\cdot f'(x)\cdot\Delta x + \underbrace{f'(x)^2(\Delta x)^2}_{\to f'(x)^2\cdot 0\cdot 0\to 0} \cancel{-f(x)^2}\right)\cdot\sqrt{\frac{1}{f'(x)^2}+1}\cdot\pi$

$= 2f(x)\cdot\cancel{f'(x)}\cdot\Delta x\cdot\pi\cdot\sqrt{\frac{1+f'(x)^2}{\cancel{f(x)^2}}} = 2\pi\cdot f(x)\cdot\sqrt{f'(x)^2+1}\cdot\Delta x$

$\Rightarrow dA = 2\pi\cdot f(x)\cdot\sqrt{f'(x)^2+1}\,dx$

$\Rightarrow A = \int_{\dots}^{\dots} dA = 2\pi\int_{x_1}^{x_2} f(x)\cdot\sqrt{f'(x)^2+1}\,dx$

$$\rightsquigarrow \boxed{A_{Mantel} = 2\pi\int_{x_1}^{x_2} f(x)\cdot\sqrt{f'(x)^2+1}\,dx}$$