Übungen in Analysis ◇ Exercices en Analyse ◇ T. I1 ◇ II / 4

Probl. 1 $\int_0^{\pi} \sin^2 x\, dx = ?$ $\quad\rightsquigarrow\quad$ (!!!!!) $\sin^2 x = 1 - \cos^2 x$

Probl. 2 $\int_1^{e} x \cdot ln(x)\, dx = ?$

Probl. 3 $\int_1^{e} x^2 \cdot ln(x)\, dx = ?$

Probl. 4 $\int_0^{\pi} x^3 \cdot \sin x\, dx = ?$

Probl. 5 Studiere: • *Etudier:*

$$\int_{z=a}^{z=b} f(z)\, dz = \int_{x=z^{-1}(a)}^{x=z^{-1}(b)} f(z(x)) \cdot \frac{dz}{dx}\, dx \qquad (\Delta z = \frac{\Delta z}{\Delta x}\, \Delta x \;\Rightarrow\; dz = \frac{dz}{dx}\, dx)$$

Sei • *Soit :* $\quad f_1(x) = 4\, x \cdot ln(x^2)$

$\rightsquigarrow \int_1^{10} f_1(x)\, dx = 2 \cdot \int_{??\ 1\ ??}^{??\ 10\ ??} 2\, x \cdot ln(x^2)\, dx = ?$ $\qquad$ (Subst. $z = x^2$)

Probl. 6 $\int_0^{5} (x^5 - 6\, x^4 + 2\, x^3 + 11\, x^2 - x + 1) \cdot (5\, x^4 - 24\, x^3 + 6\, x^2 + 22\, x - 1)\, dx = ?$

Probl. 7 (a) $\int_1^{\infty} \frac{1}{x^2}\, dx = \lim_{z \to \infty} \int_1^{z} x^{-2}\, dx = ?$

(b) $\int_1^{\infty} \frac{1}{x}\, dx = ?$

(c) $\int_1^{\infty} \frac{1}{x^3}\, dx = ?$

Probl. 8 $\int_0^{1} e^{2 \cdot x^2} \cdot x^3\, dx = ?$

Probl. 9 $\int_1^{2} ln(x) \cdot \sinh(x),\, dx = ?$